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EFFECT OF MAGNETIC FIELD AND POROUS MEDIUM ON UNSTEADY FLOW OF SECOND ORDER OLDROYD VISCO-ELASTIC LIQUID BETWEEN TWO PARALLEL OSCILLATING FLAT PLATES

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Abstract

The aim of the present paper is to study the oscillatory motion of a conducting visco-elastic Oldroyd liquid of second order between two parallel flat plates through porous medium under the influence of a uniform magnetic field applied perpendicularly to the flat plates. Both the plates are considered to be oscillating harmonically with different amplitudes and different frequencies. Some particular cases have also been discussed in detail.

Keywords- *MHD, Magnetic field, porous medium, Non-Newtonian fluid, Oldroyd fluid of second order, Visco-elastic fluid.*

INTRODUCTION

The visco-elastic fluids are particular cases of non-Newtonian fluids which exhibit appreciable elastic behaviour and stress-strain velocity relations and are time dependent. The subject of rheology is of great technological importance as in many branches of industry, the problem arises of designing apparatus to transport or to process substances which cannot be governed by the classical stress-strain velocity relations. Examples of such substances and the process are many, the extrusion of plastics, in the manufacture of rayon, nylon or other textiles, fibres, visco-elastic effects are encountered when the spinning solutions and in the manufacture of lubricating greases and rubbers.

The magnetohydrodynamic flow means, the motion of the electrically conducting fluid in the presence of Maxwell electro-magnetic field. The flow of the conducting fluid is effectively changed by the presence of the magnetic field and the magnetic field is also perturbed due to the motion of the conducting fluid. This phenomenon is, therefore, interlocking in character and the discipline of this branch of science is called Magneto hydrodynamics and, in short, written as MHD. It is equally rich and admits wider applications in Engineering, Technology, Astrophysics and other applied sciences. It has tremendously developed in recent years and some of the monographs in this direction are due to Ferraro and Plumption[7]; Pai[12] ; Shercliff[14]; Sutton and Sherman[15]; Jefferey[9] and Cowling[5] etc.

Many researchers have paid their attention towards the application of visco-elastic fluid flow of different category under the influence of magnetic field through channels of various cross-sections such as Choubey[4]; Yadav and Singh[18]; Suverna and Venkataramana[16]; Kundu and Sengupta[11]; Sengupta and Paul[13]; Krishna and Rao[10]; Ghosh and Ghosh[8]; Banerjee[2]; Bodosa[3]; Devika, Satya Narayana and Venkataramana[6]; Agrawal, Agrawal and Varshney[1]; Tripathi, Sharma and Singh[17] etc.

The aim in the present paper is to discuss the oscillatory motion of a conducting visco-elastic Oldroyd liquid of second order between two parallel flat plates through porous medium and uniform magnetic field has been applied perpendicularly to the flat plates. Both the plates are considered to be oscillating harmonically with different amplitudes and different frequencies. Some particular cases have also been discussed in detail.

BASIC THEORY FOR SECOND ORDER OLDROYD VISCO-ELASTIC LIQUID

For slow motion, the rheological equations for second order Oldroyd visco-elastic liquid are:

$$\left.\begin{aligned} &\tau_{ij} = -p\delta_{ij} + \tau'_{ij} \\ &\left(1 + \lambda_1 \frac{\partial}{\partial t} + \lambda_2 \frac{\partial^2}{\partial t^2}\right)\tau'_{ij} = 2\mu\left(1 + \mu_1 \frac{\partial}{\partial t} + \mu_2 \frac{\partial^2}{\partial t^2}\right)e_{ij} \quad \dots(1) \\ &e_{ij} = \frac{1}{2}(v_{i,j} + v_{j,i}) \end{aligned}\right\}$$

where τ_{ij} is the stress tensor, τ'_{ij} the deviatoric stress tensor, e_{ij} the rate of strain tensor , *p* the fluid pressure , λ_1 the stress relaxation time parameter, μ_1 the strain rate retardation time parameter, λ_2 the additional material constant, μ_2 the additional material constant, δ_{ij} the metric tensor, μ the coefficient of viscosity and v_i is the velocity components.

FORMULATION OF THE PROBLEM

Let there be considered *d* be the distance between the parallel flat plates, *x*-axis along the lower plate in the direction of flow of liquid and *y*-axis along the perpendicular to the plates. Suppose that the lower plate and upper plate execute oscillations with different amplitudes v_1, v_2 and different frequencies ω_1 , ω_2 respectively.

Following the stress-strain relations, the equation of motion for visco-elastic Oldroyd liquid of second order between two oscillating flat plates through porous medium and under the influence of uniform magnetic field applied perpendicularly to the plates, when induced magnetic effect is neglected, is given by

$$\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)\frac{\partial u}{\partial t}=-\frac{1}{\rho}\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)\frac{\partial p}{\partial x}$$
$$+\nu\left(1+\mu_1\frac{\partial}{\partial t}+\mu_2\frac{\partial^2}{\partial t^2}\right)\frac{\partial^2 u}{\partial y^2}-\left(\frac{\sigma B_0^2}{\rho}+\frac{1}{K}\right)\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)u \quad \ldots(2)$$

where u is the velocity of visco-elastic liquid in the direction of oscillation, p the fluid pressure, t the time, ν the kinetic viscosity, σ the electrical conductivity, B_0 is the magnetic inductivity and K is the coefficient of permeability of porous medium .

Assuming that the pressure gradient is zero, the equation (2) becomes

$$\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)\frac{\partial u}{\partial t}=\nu\left(1+\mu_1\frac{\partial}{\partial t}+\mu_2\frac{\partial^2}{\partial t^2}\right)\frac{\partial^2 u}{\partial y^2}$$
$$-\left(\frac{\sigma B_0^2}{\rho}+\frac{1}{K}\right)\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)u \quad \ldots(3)$$

The boundary conditions are:

$$\left.\begin{aligned} u&=V_1e^{-i\omega_1 t}, \quad \text{when } y=0\\ u&=V_2e^{-i\omega_2 t}, \quad \text{when } y=d\end{aligned}\right\} \quad \ldots(4)$$

Introducing the following non-dimensional quantities:

$$y^*=\frac{1}{d}y\,,\ \ u^*=\frac{d}{\nu}u\,,\ \ t^*=\frac{\nu}{d^2}t\,,\ \ \omega_1^*=\frac{d^2}{\nu}\omega_1\ \ ,\omega_2^*=\frac{d^2}{\nu}\omega_2\,,$$
$$\lambda_1^*=\frac{\nu}{d^2}\lambda_1\,,\ \ \mu_1^*=\frac{\nu}{d^2}\mu_1\,,\ \ \lambda_2^*=\frac{\nu^2}{d^4}\lambda_2\,,\ \ \mu_2^*=\frac{\nu^2}{d^4}\mu_2$$

in the equations (3) and (4), it is found (after dropping the stars)

$$\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)\frac{\partial u}{\partial t}=\left(1+\mu_1\frac{\partial}{\partial t}+\mu_2\frac{\partial^2}{\partial t^2}\right)\frac{\partial^2 u}{\partial y^2}$$
$$-\left(H^2+\frac{1}{K}\right)\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)u \quad \ldots(5)$$

where $H=B_0d\sqrt{\sigma/\mu}$ (Hartmann number)

and boundary conditions are:

$$\left.\begin{aligned} u&=V_1e^{-i\omega_1 t}, \quad \text{when } y=0\\ u&=V_2e^{-i\omega_2 t}, \quad \text{when } y=1\end{aligned}\right\} \quad \ldots(6)$$

SOLUTION OF THE PROBLEM

For the solution of equation (5), it may be taken

$$u = v_1 f(y) e^{-i\omega_1 t} + v_2 g(y) e^{-i\omega_2 t} \qquad \dots (7)$$

which is evidently periodic in t

Substituting equation (7) in equation (5), it is found

$$v_1 e^{-i\omega_1 t}\left[(1 - i\omega_1\mu_1 - \omega_1^2\mu_2)\frac{d^2 f}{dy^2} + \{i\omega_1(1 - i\omega_1\lambda_1 - {\omega_1}^2\lambda_2)\right.$$

$$\left. -\left(H^2 + \frac{1}{K}\right)(1 - i\omega_1\lambda_1 - {\omega_1}^2\lambda_2)\right\} f(y)\Big] + v_2 e^{-i\omega_2 t}\left[(1 - i\omega_2\mu_1 - \omega_2^2\mu_2)\frac{d^2 g}{dy^2}\right.$$

$$+\left\{i\omega_2(1 - i\omega_2\lambda_1 - {\omega_2}^2\lambda_2) - \left(H^2 + \frac{1}{K}\right)(1 - i\omega_2\lambda_1 - {\omega_2}^2\lambda_2)\right\} g(y)] = 0$$

or

$$v_1 e^{-i\omega_1 t}(1 - i\omega_1\mu_1 - \omega_1^2\mu_2)\left[\frac{d^2 f}{dy^2} + \frac{(1 - i\omega_1\lambda_1 - {\omega_1}^2\lambda_2)\left\{i\omega_1 - \left(H^2 + \frac{1}{K}\right)\right\}}{(1 - i\omega_1\mu_1 - \omega_1^2\mu_2)} f(y)\right]$$

$$+v_2 e^{-i\omega_2 t}(1 - i\omega_2\mu_1 - {\omega_2}^2\mu_2)\left[\frac{d^2 g}{dy^2} + \frac{(1 - i\omega_2\lambda_1 - {\omega_2}^2\lambda_2)\left\{i\omega_2 - \left(H^2 + \frac{1}{K}\right)\right\}}{(1 - i\omega_2\mu_2 - {\omega_2}^2\mu_2)} g(y)\right]$$

$$= 0$$

or

$$v_1 e^{-i\omega_1 t}(1 - i\omega_1\mu_1 - {\omega_1}^2\mu_2)\left[\frac{d^2 f}{dy^2} + m^2 f\right]$$

$$+v_2 e^{-i\omega_2 t}(1 - i\omega_2\mu_1 - {\omega_2}^2\mu_2)\left[\frac{d^2 g}{dy^2} + n^2 g\right] = 0 \qquad \dots (8)$$

where $m^2 = \dfrac{(1 - i\omega_1\lambda_1 - {\omega_1}^2\lambda_2)\left\{i\omega_1 - \left(H^2 + \frac{1}{K}\right)\right\}}{(1 - i\omega_1\mu_1 - {\omega_1}^2\mu_2)}$

and $n^2 = \dfrac{(1 - i\omega_2\lambda_1 - {\omega_2}^2\lambda_2)\left\{i\omega_2 - \left(H^2 + \frac{1}{K}\right)\right\}}{(1 - i\omega_2\mu_2 - {\omega_2}^2\mu_2)}$

By assumption v_1 and v_2 can not be zero, therefore from equation (8), it is found

$$\frac{d^2 f}{dy^2} + m^2 f = 0 \qquad \dots (9)$$

$$\frac{d^2 g}{dy^2} + n^2 g = 0 \qquad \dots (10)$$

From equations (6) and (7), the corresponding boundary conditions reduced to

$$\left.\begin{array}{l} f(y) = 1, \;\; g(y) = 0, \;\; \text{when } y = 0 \\ f(y) = 0, \;\; g(y) = 1, \;\; \text{when } y = 1 \end{array}\right\} \qquad \dots(11)$$

Solutions of equations (9) and (10), subject to boundary conditions given by equation (11), are

$$f(y) = \frac{\sin m\,(1-y)}{\sin m} \qquad \dots(12)$$

and $$g(y) = \frac{\sin ny}{\sin n} \qquad \dots(13)$$

Now substituting the values of *f(y)* and *g(y)* in equation (7), the velocity of second order Oldroyd visco-elastic liquid between two oscillating flat parallel plates through porous medium and under the influence of a uniform magnetic field applied perpendicularly to the plates is obtained

$$u = v_1 \frac{\sin m\,(1-y)}{\sin m} e^{-i\omega_1 t} + v_2 \frac{\sin ny}{\sin n} e^{-i\omega_2 t} \qquad \dots(14)$$

PARTICULAR CASES

CASE I: If material constants λ_2 and μ_2 both are zero i.e. $\lambda_2 = 0$ and $\mu_2 = 0$

Then from equation (14), velocity of Oldroyd visco-elastic liquid between two oscillating flat plates under the influence of a uniform magnetic field through porous medium is found.

$$u = v_1 \frac{\sin m\,(1-y)}{\sin m} e^{-i\omega_1 t} + v_2 \frac{\sin ny}{\sin n} e^{-i\omega_2 t} \qquad \dots(15)$$

where $$m = \left[\frac{(1 - i\omega_1\lambda_1)\left\{i\omega_1 - \left(H^2 + \frac{1}{K}\right)\right\}}{(1 - i\omega_1\mu_1)}\right]^{\frac{1}{2}}$$

$$n = \left[\frac{(1 - i\omega_2\lambda_1)\left\{i\omega_2 - \left(H^2 + \frac{1}{K}\right)\right\}}{(1 - i\omega_2\mu_1)}\right]^{\frac{1}{2}}$$

CASE II: If both the flat plates oscillate with same amplitude and different frequencies i.e. $v_1 = v_2 = v$ (say)

Then from equation (14), it is found

$$u = v\left[\frac{\sin m\,(1-y)}{\sin m} e^{-i\omega_1 t} + \frac{\sin ny}{\sin n} e^{-i\omega_2 t}\right] \qquad \dots(16)$$

CASE III: If both the flat plates oscillate with same frequencies and different amplitudes i.e. $\omega_1 = \omega_2 = \omega$ (say)

Then from equation (14), it is found

$$u = \left[v_1 \frac{\sin m\,(1-y)}{\sin m} + v_2 \frac{\sin ny}{\sin n}\right] e^{-i\omega t} \quad \dots (17)$$

where $$m = n = \left[\frac{(1 - i\omega\lambda_1 - \omega^2\lambda_2)\left\{i\omega - \left(H^2 + \frac{1}{K}\right)\right\}}{(1 - i\omega\mu_1 - \omega^2\mu_2)}\right]^{\frac{1}{2}} e^{-i\omega t}$$

CASE IV: If both the flat plates oscillate with same amplitude and same frequency i.e. $v_1 = v_2 = v$ (say) and $\omega_1 = \omega_2 = \omega$ (say)

Then from equation (14), it is found

$$u = v\{(1 - \cot m)\sin my + \cos my\}e^{-i\omega t} \quad \dots (18)$$

where $$m = \left[\frac{(1 - i\omega\lambda_1 - \omega^2\lambda_2)\left\{i\omega - \left(H^2 + \frac{1}{K}\right)\right\}}{(1 - i\omega\mu_1 - \omega^2\mu_2)}\right]^{\frac{1}{2}}$$

CASE V: If magnetic field and porous medium both are withdrawn i.e. $H = 0$, $K \to \infty$

Then from equation (14), it is found

$$u = v_1 \frac{\sin m\,(1-y)}{\sin m} e^{-i\omega_1 t} + v_2 \frac{\sin ny}{\sin n} e^{-i\omega_2 t} \quad \dots (19)$$

where $$m = \left[\frac{i\omega_1(1 - i\omega_1\lambda_1 - {\omega_1}^2\lambda_2)}{(1 - i\omega_1\mu_1 - {\omega_1}^2\mu_2)}\right]^{\frac{1}{2}}$$

$$n = \left[\frac{i\omega_2(1 - i\omega_2\lambda_1 - {\omega_2}^2\lambda_2)}{(1 - i\omega_2\mu_1 - {\omega_2}^2\mu_2)}\right]^{\frac{1}{2}}$$

CONCLUSION

The tendency of uniform magnetic field applied perpendicularly to the flat plates through porous medium is to reduce the velocity of visco-elastic liquid of different category.

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